Basic properties: $d \equiv \, ^2_1\mathrm{H}$

mass: mc^2 = 1876.124 MeV

binding energy: $B \equiv \sum_i m_i - M \;=\; m_p + m_n - m_d \;=\; 2.2245731 \;\text{MeV}$

(measured via γ-ray energy in n + p → d + γ)

RMS radius: 1.963 ± 0.004 fm (from electron scattering)

quantum numbers: $J^\pi, I \;=\; 1^+,\; 0$ (lectures 13, 14)

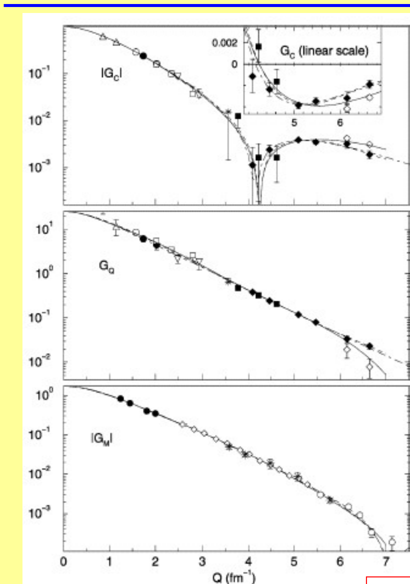
magnetic moment: μ = + 0.8573 μ_N

electric quadrupole moment: Q = + 0.002859 ± 0.00030 bn

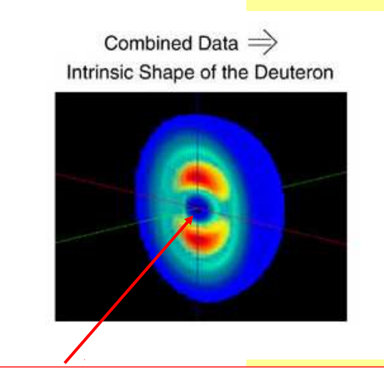
(→ the deuteron is not spherical! ....)

Important because:

- deuterium is the lightest nucleus and the only bound N-N state
- testing ground for state-of-the art models of the N-N interaction.



Because the deuteron has spin 1, there are 3 form factors to describe elastic scattering: the "charge" (G_c), "electric quadrupole" (G_q) and "magnetic (G_M) form factors. (JLab data)



Interesting feature: strong attractive np force, but a void in the center – the deuteron is hollow! ... Why?

Interpretation of quantum numbers: $J^{\pi}, I = 1^{+},\ 0$

3

$$\vec{S} \equiv \vec{S}_n + \vec{S}_p$$

$$\vec{J} = \vec{S} + \vec{L} = \vec{1}$$

$$\pi = (+)\,(+)\,(-1)^L = + \Rightarrow L = 0, 2, 4 \ .....$$

- Of the possible quantum numbers, L = 0 has the lowest energy, so we expect the ground state to be L = 0, S = 1 (the deuteron has no excited states!)
- The nonzero electric quadrupole moment suggests an admixture of L = 2 (more later!)

introduce **Spectroscopic Notation:** $^{2S+1}L_J$

with naming convention: L = 0 is an S-state, L = 1 is a P-state, L = 2: D-state, etc...

$\rightarrow$ the deuteron configuration is primarily $^3S_1$

Isospin and the N-N system:

4

$$\vec{I} = \vec{\tfrac{1}{2}} + \vec{\tfrac{1}{2}} \Rightarrow I = 0,\ 1$$

The total wavefunction for two identical Fermions has to be antisymmetric w.r.to particle exchange:

$$\Psi_{total} = \psi_{space} \times \phi_{spin} \times \chi_{isospin}$$

Central force problem:

$$\psi_{space}(r,\theta,\phi) = f(r)\ Y_{LM}(\theta,\phi)$$

with symmetry $(-1)^L$ given by the spherical harmonic functions

Spin and Isospin configurations:

S = 0 and I = 0 are antisymmetric ;

S = 1 and I = 1 are symmetric

$^3S_1$ state can only be I = 0 !

Magnetic Moment: $\mu_d = 0.857\ \mu_N$

5

In general, the magnetic moment is a quantum-mechanical vector; it must be aligned along the "natural symmetry axis" of the system, given by the total angular momentum:

$$\vec{\mu} \sim \vec{J}$$

But we don't know the direction of $\vec{J}$, only its "length" and z-projection as expectation values:

$$\langle J^2 \rangle = J(J+1); \qquad \langle J_z \rangle = m_J = (-J....+J)$$

in a magnetic field, the energy depends on m_J via $\Delta E = -\langle \vec{\mu} \bullet \vec{B} \rangle \equiv -g_J\, m_J\, B\, \mu_N$

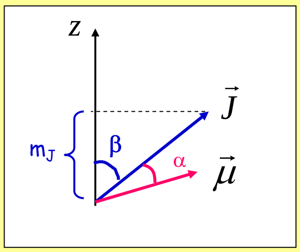
Strategy: we will define the magnetic moment by its maximal projection on the z-axis, defined by the direction of the magnetic field, with $m_J = J$

$$\mu \equiv \langle \vec{\mu} \bullet \hat{z} \rangle \big|_{m_J = J} = g_J\, J\, \mu_N$$

Use expectation values of operators to calculate the result.....

Calculation of μ :

6



$$\mu \equiv \langle \vec{\mu} \bullet \hat{z} \rangle \big|_{m_J = J} = g_J\, J\, \mu_N$$

Subtle point: we have to make two successive projections to evaluate the magnetic moment according to our definition, and the spin and orbital contributions enter with different weights.

1. Project onto the direction of J: $\vec{\mu} \bullet \hat{J} = \mu \cos\alpha = \dfrac{\vec{\mu} \bullet \vec{J}}{\sqrt{J(J+1)}}$

2. Project onto the z-axis with $m_J = J$: $\cos\beta = \dfrac{m_J}{|J|} = \dfrac{J}{\sqrt{J(J+1)}}$

$$\mu \equiv g_J\, J\, \mu_N = \mu \cos\alpha \cos\beta = \langle \vec{\mu} \bullet \vec{J} \rangle \frac{1}{(J+1)}$$

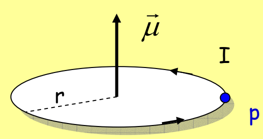
Next, we need to figure out the operator for $\vec{\mu}$

spin and orbital contributions: 7

We already know the intrinsic magnetic moments of the proton and neutron, so these must correspond to the spin contributions to the magnetic moment operator:

$$\mu_p = +2.79\,\mu_N = g_{s,p}\; S\;\; \mu_N \;\; \Rightarrow \;\; g_{s,p} = +5.58$$
$$\mu_n = -1.91\,\mu_N = g_{s,n}\; S\;\; \mu_N \;\; \Rightarrow \;\; g_{s,n} = -3.83$$

For the orbital part, there is a contribution from the proton only, corresponding to a circulating current loop (semiclassical sketch, but the result is correct)



$$\vec{\mu} = I\,\pi\, r^2\,\hat{\ell} = g_\ell\,\vec{\ell}\,\mu_N\,, \quad g_\ell = 1$$

For the deuteron, we want to use the magnetic moment operator:

$$\vec{\mu} = \left(g_{s,p}\,\vec{S}_p + g_{s,n}\,\vec{S}_n + \vec{L}_p\right)\mu_N$$

Details: 8

1. $\vec{L}_p = \frac{1}{2}\,\vec{L}$ because $m_n \cong m_p$, and L is the total orbital angular momentum !

2. L = 0 in the "S-state" ($^3S_1$) but we will consider also a contribution from the "D-state" ($^3D_1$) as an exercise

3. The proton and neutron couple to S = 1, and the deuteron has J = 1

$$\mu = \tfrac{1}{2}\left\langle \vec{\mu}\bullet\vec{J}\right\rangle = \tfrac{1}{2}\left\langle \left(g_{s,p}\,\vec{S}_p + g_{s,n}\,\vec{S}_n + \tfrac{1}{2}\vec{L}\right)\bullet\vec{J}\right\rangle \mu_N$$

Trick: use $\vec{S}_n + \vec{S}_p = \vec{S}$ and write the **operator** as:

$$\vec{\mu} = \left(\tfrac{1}{2}\,(g_{s,p}+g_{s,n})\,\vec{S} + \cancel{\tfrac{1}{2}\,(g_{s,p}-g_{s,n})\,(\vec{S}_p-\vec{S}_n)} + \tfrac{1}{2}\,\vec{L}\right)\mu_N$$

But the proton and neutron spins are aligned, and $\left\langle \vec{S}_p\bullet\vec{J}\right\rangle = \left\langle \vec{S}_n\bullet\vec{J}\right\rangle$

so the second term has to give zero!

continued.... 9

So, effectively we can write for the deuteron:

$$\mu = \tfrac{1}{4} \left\langle \left((g_{s,p} + g_{s,n}) \ \vec{S} + \vec{L} \right) \bullet \vec{J} \right\rangle \ \mu_N$$

Trick for expectation values:

$$\vec{J} = \vec{L} + \vec{S} \, ; \quad \left\langle J^2 \right\rangle = J\,(J+1)$$

$$\left\langle J^2 \right\rangle = \left\langle (\vec{L}+\vec{S}) \bullet (\vec{L}+\vec{S}) \right\rangle = \left\langle L^2 + S^2 + 2\vec{L} \bullet \vec{S} \right\rangle$$

$$\left\langle \vec{S} \bullet \vec{J} \right\rangle = \left\langle \vec{S} \bullet \vec{L} + \vec{S} \bullet \vec{S} \right\rangle = \tfrac{1}{2} \left\langle J^2 - L^2 - S^2 \right\rangle + \left\langle S^2 \right\rangle = \tfrac{1}{2} \left\langle J^2 - L^2 + S^2 \right\rangle$$

$$\left\langle \vec{L} \bullet \vec{J} \right\rangle = \left\langle \vec{L} \bullet \vec{L} + \vec{L} \bullet \vec{S} \right\rangle = \tfrac{1}{2} \left\langle J^2 + L^2 - S^2 \right\rangle$$

$$\mu\,(^3S_1) = \tfrac{1}{2} (g_{s,p} + g_{s,n}) \ \mu_N = \mu_p + \mu_n = 0.880 \ \mu_N$$

$$\mu\,(^3D_1) = \tfrac{1}{4} \left(3 - (g_{s,p} + g_{s,n}) \right) \mu_N = 0.310 \ \mu_N$$

Comparison to experiment: 10

$$\mu_d = 0.857 \ \mu_N$$

This is intermediate between the S-state and D-state values:

$$\mu\,(^3S_1) = 0.880 \ \mu_N$$

$$\mu\,(^3D_1) = 0.310 \ \mu_N$$

Suppose the wave function of the deuteron is a linear combination of S and D states:

$$\left| \psi_d \right\rangle = a \left| ^3S_1 \right\rangle + b \left| ^3D_1 \right\rangle \quad \text{with} \quad a^2 + b^2 = 1$$

Then we can adjust the coefficients to explain the magnetic moment:

$$\mu_d = (1 - b^2) \ \mu\,(^3S_1) + b^2 \ \mu\,(^3D_1)$$

$b^2 = 0.04$, or a 4% D-state admixture accounts for the magnetic moment !

Deuteron Summary so far

11

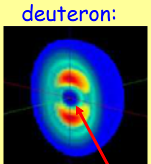
Quantum numbers: $(J^\pi, I) = (1^+, 0)$ favor a $^3S_1$ configuration (with S = 1, L = 0) as the lowest energy n-p bound state

Magnetic moment: $\mu = 0.857\ \mu_N$ is 2.6% smaller than for a pure $^3S_1$ state and is consistent with a linear combination of L = 0 and L = 2 components:

$$|\psi_d\rangle = a\ |^3S_1\rangle + b\ |^3D_1\rangle \quad with \quad a^2 + b^2 = 1 \quad and \quad b^2 = 0.04$$

If this is correct, we can draw two conclusions about the N-N force:

deuteron:



1. The S = 1 configuration has lowest energy (i.e., observed deuteron quantum numbers) , so the N-N potential must be more attractive for total spin S = 1 than for S = 0.

2. The lowest energy solution for a spherically symmetric potential is purely L = 0, and the L = 2 wave functions are orthogonal to L = 0 wave functions, so a physical deuteron state with mixed symmetry can only arise if the N-N potential is not exactly spherically symmetric!

Also: the "hole" in the middle of the deuteron means that at very short distances the N-N potential must be repulsive – the neutron and proton do not overlap!

Deuteron Quadrupole moment etc…

12

Formalism for electrostatic moments:

(*see also D.J. Griffiths, `Introduction to Electodynamics' Ch. 3, and 16.369)*

- Recall that an electric charge distribution of arbitrary shape can be described by an infinite series of multipole moments (spherical harmonic expansion), with terms of higher L reflecting more complicated departures from spherical symmetry
- The energy of such a distribution interacting with an external electric field $\vec{E} = -\vec{\nabla} V$ can be written as:

$$E_{\text{int}} = V(0)\ q\ +\ \left.\frac{\partial V}{\partial z}\right|_0 p_z\ -\frac{1}{4}\left.\frac{\partial^2 V}{\partial z^2}\right|_0 Q_{zz}\ +\ \ldots$$

where the charge distribution is located at the origin and has a symmetry axis along z;

q is the electric charge or `monopole moment'

p_z is the electric dipole moment

Q_{zz} is the electric quadrupole moment

and in general, higher order multipole moments couple to higher derivatives of the external potential at the origin.

Formalism, continued.... (axially symmetric case) 13

The multipole moments are defined in general by: $E_\ell \;=\; \int \rho(\vec{r})\, \hat{E}_\ell \; d^3 r$

with corresponding multipole operators proportional to spherical harmonic functions of order ℓ: $\hat{E}_\ell \;=\; c_\ell \; r^\ell \; Y_{\ell 0}(\theta)$

The first few multipole operators and associated moments are:

order, ℓ	Moment	Symbol	operator, $\hat{E}_\ell$
0	*monopole*	q	$1 = \sqrt{4\pi}\; r^0\; Y_{00}(\theta)$
1	*dipole*	p_z	$z = r\cos\theta = \frac{\sqrt{4\pi}}{3}\; r^1\; Y_{10}(\theta)$
2	*quadrupole*	Q_{zz}	$r^2(3\cos^2\theta - 1) = \frac{\sqrt{16\pi}}{5}\; r^2\; Y_{20}(\theta)$

Connection to nuclei and the deuteron problem: 14

For a nuclear system, the multipole moments are expectation values of the multipole operators, e.g. the electric charge:

$$q \;\equiv\; \int \rho(\vec{r})\, d^3 r \;\equiv\; e \sum_{i=1}^{Z} \int \psi_i^*(\vec{r})\, \hat{E}_0\, \psi_i(\vec{r})\, d^3 r \;=\; +Ze$$

The electric charge density is proportional to the probability density, i.e. the wave function squared, summed up for all the protons in the system !!!

We already know the electric charge of the deuteron, but what about higher moments?

Electric dipole moment, general case:

integral **must** vanish!

$$p_z \;=\; \int \Psi^* \hat{E}_1\, \Psi\, d^3 r \;=\; \int |\Psi|^2\, r\, \boxed{\cos\theta}\; d^3 r \;=\; 0$$

total wave function of the system

even under space reflection

odd, $\ell = 1$

continued... 15

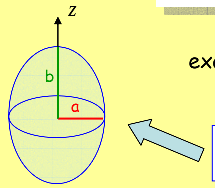
Basic symmetry property: no quantum system can have an electric dipole moment if its wave function has definite parity, i.e. if $|\Psi|^2$ is even.

(Incidentally, the possibility of a nonzero electric dipole moment of the neutron is of great current interest: so far the measured upper limit is $|p_n| < 10^{-25}$ e – cm ....)

In fact, all odd multipole moments must vanish for the same reason

First nontrivial case: Electric Quadrupole Moment Q_{zz} *(F&H sec. 18.1)*

$$Q_{zz} = \int \rho(\vec{r})\, r^2\, (3\cos^2\theta - 1)\, d^3r$$



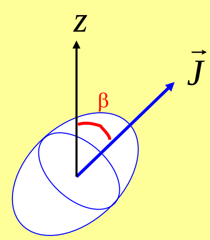
example: uniform density object with ellipsoidal shape

$$Q_{zz} = \frac{2}{5}\, Ze\,(b^2 - a^2)$$

Q > 0 if b > a: "prolate"
Q < 0 if b < a: "oblate"

How to relate this to a nucleus, e.g. the deuteron? 16

Assume that the charge distribution is an ellipsoid of revolution with symmetry axis along the total angular momentum vector J:



As for the magnetic dipole moment, we specify the **intrinsic** electric quadrupole moment as the expectation value when J is maximally aligned with z:

$$Q_{\text{int}} = \left\langle \hat{E}_2 \right\rangle \Big|_{m_J = J}$$

"Quantum geometry": $\cos\beta = \frac{m_J}{|J|} = \frac{J}{\sqrt{J(J+1)}}$

If an electric field gradient is applied along the z-axis as shown, the observable energy will shift by an amount corresponding to the intrinsic quadrupole moment transformed to a coordinate system rotated through angle β to align with the z-axis

Result: 17

Let Q_{lab} be the electric quadrupole moment we **measure** for the ellipsoidal charge distribution with $m_J = J$:

$$Q_{\text{lab}} = \underbrace{\frac{1}{2}\left(3\cos^2\beta - 1\right)}_{\text{standard, classical prescription for rotated coordinates}} Q_{\text{int}} = \underbrace{\left(\frac{J-1/2}{J+1}\right)}_{\text{"Quantum geometry" for the rotation function}} Q_{\text{int}}$$

How do we apply this to anything?

1. A spherically symmetric state (L = 0) has Q_{int} = 0 (e.g. deuteron S-state)
2. Even a distorted state with J = ½ will not have an observable quadrupole moment
3. J = 1, L = 2 is the smallest value of total angular momentum for which we can observe a nonzero quadrupole moment

→ ***these are the quantum numbers for the deuteron D-state!***

Deuteron intrinsic quadrupole moment: 18

recall our model for the deuteron wave function:

$$|\psi_d\rangle = a\,|{}^3S_1\rangle + b\,|{}^3D_1\rangle$$

result for the quadrupole moment:

note cancellation here

$$Q_{\text{int}} = \frac{\sqrt{2}}{10}\,|a^* b|\,\langle r^2\rangle_{SD} - \frac{1}{20}\,b^2\,\langle r^2\rangle_{DD}$$

$$= +0.00286 \pm 0.00003 \text{ bn}$$

A good model of the N-N interaction can fit both the magnetic moment and the quadrupole moment of the deuteron with the same values of a and b! →

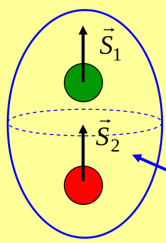
Basic features of the N-N potential, via the deuteron, etc.: 19

1. Independent of the value of L, the state with intrinsic spins coupled to S = 1 has lower energy

   → this implies a term proportional to:

$$-\left\langle \vec{S}_1 \bullet \vec{S}_2 \right\rangle = -\tfrac{1}{2}\left\langle S^2 - S_1^2 - S_2^2 \right\rangle = \begin{cases} -1/4, & S = 1 \\ +3/4, & S = 0 \end{cases}$$

2. The deuteron quadrupole moment implies a non-central component, i.e. the potential is not spherically symmetric. Since the symmetry axis for Q is along J, Q > 0 means that the matter distribution is stretched out along the J – axis:



→ This implies a "tensor" force, proportional to:

$$-\left\langle S_{12} \right\rangle = -\left\langle 3\,\frac{(\vec{S}_1 \bullet \vec{r})\,(\vec{S}_2 \bullet \vec{r})}{r^2} - \vec{S}_1 \bullet \vec{S}_2 \right\rangle$$

compare: magnetic dipole-dipole interaction, see Griffiths problem 6.20

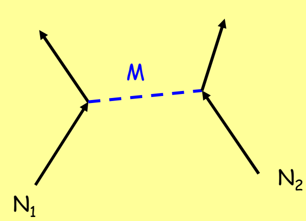
N-N interaction continued.... 20

3. There is also a spin-orbit term, as deduced from N-N scattering experiments with a polarized beam:

$$V_{S-O} \sim \left\langle \vec{L} \bullet \vec{S} \right\rangle$$

(This plays a very important role also in determining the correct order of energy levels in nuclear spectra – more later!)

4. Finally, all contributions to the N-N interaction are based on a microscopic meson exchange mechanism, as explored in assignment 5:



Where M is a π, ρ, ω ... meson, etc.

and each term has a spatial dependence of the form:

$$V(r) = g\,\frac{e^{-m\,r}}{r} \times (spin\ \ function)$$

State of the art N-N interaction model: *(not to scare anybody...)* 21

PHYSICS REPORTS (Review Section of Physics Letters) 149, No. 1 (1987) 1–89. North-Holland, Amsterdam

(89 page exposition of one of only ~3 state-of-the-art models of the N-N interaction worldwide – constantly refined and updated since first release.)

THE BONN MESON-EXCHANGE MODEL FOR THE NUCLEON–NUCLEON INTERACTION*

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Brace yourself – it takes 2 pages just to write all the terms down!!! 22

pseudoscalar mesons:

(tensor operator)

$$V_{\text{ps}}(m_{\text{ps}}, r) = \frac{1}{12}\frac{g_{\text{ps}}^2}{4\pi} m_{\text{ps}}\left[\left(\frac{m_{\text{ps}}}{m}\right)^2 Y(m_{\text{ps}}r)\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + Z(m_{\text{ps}}r)\, S_{12}\right]; \tag{F.6}$$

(they use "σ" for spin)

scalar mesons:

$$V_{\text{s}}(m_{\text{s}}, r) = -\frac{g_{\text{s}}^2}{4\pi} m_{\text{s}}\left\{\left[1 - \frac{1}{4}\left(\frac{m_{\text{s}}}{m}\right)^2\right] Y(m_{\text{s}}r) + \frac{1}{4m^2}\left[\nabla^2 Y(m_{\text{s}}r) + Y(m_{\text{s}}r)\nabla^2\right] + \tfrac{1}{2} Z_1(m_{\text{s}}r)\, \boldsymbol{L}\cdot\boldsymbol{S}\right\}; \tag{F.7}$$

(spin-orbit interaction)

vector mesons:

$$\begin{aligned}V_{\text{v}}(m_{\text{v}}, r) = \frac{g_{\text{v}}^2}{4\pi} m_{\text{v}}\Big\{&\left[1 + \frac{1}{2}\left(\frac{m_{\text{v}}}{m}\right)^2\right] Y(m_{\text{v}}r) - \frac{3}{4m^2}\left[\nabla^2 Y(m_{\text{v}}r) + Y(m_{\text{v}}r)\nabla^2\right] \\ &+ \frac{1}{6}\left(\frac{m_{\text{v}}}{m}\right)^2 Y(m_{\text{v}}r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - \tfrac{3}{2} Z_1(m_{\text{v}}r)\boldsymbol{L}\cdot\boldsymbol{S} - \tfrac{1}{12} Z(m_{\text{v}}r) S_{12}\Big\} \\ &+ \frac{1}{2}\frac{g_{\text{v}} f_{\text{v}}}{4\pi} m_{\text{v}}\{(m_{\text{v}}/m)^2\, Y(m_{\text{v}}r) + \tfrac{2}{3}(m_{\text{v}}/m)^2\, Y(m_{\text{v}}r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \\ &- 4Z_1(m_{\text{v}}r)\boldsymbol{L}\cdot\boldsymbol{S} - \tfrac{1}{3} Z(m_{\text{v}}r) S_{12}\} \\ &+ \frac{f_{\text{v}}^2}{4\pi} m_{\text{v}}\{\tfrac{1}{6}(m_{\text{v}}/m)^2\, Y(m_{\text{v}}r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - \tfrac{1}{12} Z(m_{\text{v}}r) S_{12}\},\end{aligned}$$

more of this... 23

with

$$Y(x) = e^{-x}/x \,, \qquad Z(x) = (m_\alpha/m)^2(1 + 3/x + 3/x^2)Y(x) \,,$$

"Yukawa functions" and derivatives...

$$Z_1(x) = \left(\frac{m_\alpha}{m}\right)^2 (1/x + 1/x^2)Y(x) \,, \qquad S_{12} = 3\,\frac{(\sigma_1 \cdot r)(\sigma_2 \cdot r)}{r^2} - \sigma_1 \cdot \sigma_2 \,, \tag{F.8}$$

and

$$\nabla^2 = +\frac{1}{r}\frac{d^2}{dr^2}r - \frac{L^2}{r^2} \,.$$

We use units such that $\hbar = c = 1$ ($\hbar c = 197.3286$ MeV fm). The use of the form factor, eq. (3.3), at each vertex (with $n_\alpha = 1$) leads to the following extended expressions:

$$V_\alpha(r) = V_\alpha(m_\alpha, r) - \frac{\Lambda_{\alpha,2}^2 - m_\alpha^2}{\Lambda_{\alpha,2}^2 - \Lambda_{\alpha,1}^2} V_\alpha(\Lambda_{\alpha,1}, r) + \frac{\Lambda_{\alpha,1}^2 - m_\alpha^2}{\Lambda_{\alpha,2}^2 - \Lambda_{\alpha,1}^2} V_\alpha(\Lambda_{\alpha,2}, r) \,, \tag{F.9}$$

where $\Lambda_{\alpha,1} = \Lambda_\alpha + \varepsilon$, $\Lambda_{\alpha,2} = \Lambda_\alpha - \varepsilon$, $\varepsilon/\Lambda_\alpha \ll 1$. $\varepsilon = 10$ MeV is an appropriate choice.

The full NN potential is the sum of the contributions from six mesons:

$$V(r) = \sum_{\alpha=\pi,\rho,\eta,\omega,\delta,\sigma} V_\alpha(r)$$

Parameters and Predictions: 24

R. Machleidt et al., The Bonn meson-exchange model for the nucleon–nucleon interaction

Table 14

Meson and low-energy parameters (LEP) for the configuration space one-boson-exchange potential (OBEPR)

	$g_\alpha^2/4\pi$; $[f_\alpha/g_\alpha]$	m_α (MeV)	Λ_α (GeV)	LEP	Theory
π	14.9	138.03	1.3	ε_d (MeV)	2.2246
				P_D (%)	4.81
ρ	0.95; [6.1]	769	1.3	Q_d (fm^2)	0.274
				μ_d (μ_N)	0.8524
η	3	548.8	1.5	A_S (fm$^{-1/2}$)	0.8860
				D/S	0.0260
ω	20; [0.0]	782.6	1.5	r_d (fm)	1.9691
				a_s (fm)	−23.751
δ	2.6713	983	2.0	r_s (fm)	2.662
				a_t (fm)	5.423
σ	7.7823 [a]	550 [a]	2.0	r_t (fm)	1.759

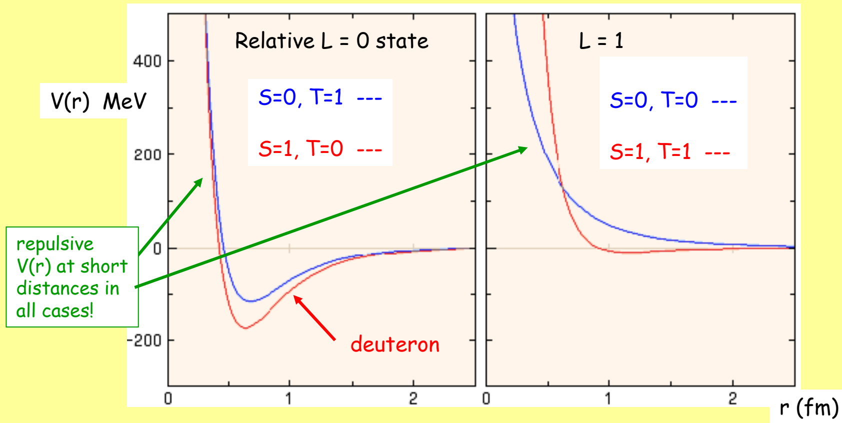
deuteron properties:

low energy scattering parameters, etc:

Impressively good agreement for ~ 10,000 experimental data points in assorted n-p and p-p scattering experiments plus deuteron observables: χ^2/d.f <~ 1.07 !

What does the NN potential look like? 25

It looks different in different spectroscopic states of the 2N system!



Only the deuteron is bound! Its quantum numbers have the deepest potential well.